Model Answer
Set- I
Std. – 10th EM/Semi
Subject – Geometry

Time : 2 Hrs. **Marks : 40**

Ans. Option d.

V + F - E = 2

2) To draw a tangent to a circle without using its centre we use ....

(a) inscribed angle theorem

(b) isosceles triangle theorem

(c) property of alternate angle test

(d) property of angles in alternate segment

Ans. Option (d)

3) The longest chord of a circle is 7.8 cm. What is the radius of the circle?

a. 3.9 b. 7.8 c. 15.6 d. 8

Ans. Option a.

Hint : $r = \frac{d}{2}$

4) Slope of a line parallel to x-axis is ....

a. Zero b. One c. Not defined d. None of these

Ans. Option a.

B) Solve the following questions. **(4)**

1) If $\sin\theta = \frac{20}{29}$ then find $\cos\theta$

Ans. $\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{20}{29}\right)^2 + \cos^2\theta = 1$

$\frac{400}{841} + \cos^2\theta = 1$

$\cos^2\theta = 1 - \frac{400}{841}$

$= \frac{441}{841}$

Taking square root of both sides.

$\cos\theta = \frac{21}{29}$

2) Show that points P(- 2, 3), Q(1, 2), R(4, 1) are collinear.

Ans. P(- 2, 3), Q(1, 2) and R(4, 1) are given points

slope of line PQ = $\frac{Y_2 - Y_1}{X_2 - X_1} = \frac{2-3}{1-(-2)} = \frac{-1}{3}$

Slope of line QR = $\frac{Y_2 - Y_1}{X_2 - X_1} = \frac{1-2}{4-1} = \frac{-1}{3}$

Slope of line PQ and line QR is equal.

But point Q lies on both the lines.

∴ Point P, Q, R are collinear.

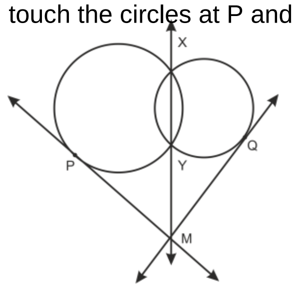
3) Ratio of corresponding sides of two similar triangles is 4:7 then find the ratio of their reas = ?

Ans. By theorem of areas of similar triangles,

Ratio of areas of the given triangles = $\left(\frac{4}{7}\right)^2 = \frac{16}{49}$ = 16:49

$\therefore$ The ratio of areas is 16:49.

4) In figure, two circles intersect each other in points X and Y. Tangents drawn from a point M on line XY touch the circles at P and Q. Prove that, seg PM ≅ seg QM.



Ans. Line MX is a common secant of the two circles.

$\therefore$ $PM^2 = MY \times MX$ ... (i)

Similarly $QM^2 = MY \times MX$, tangent secant segment theorem... (ii)

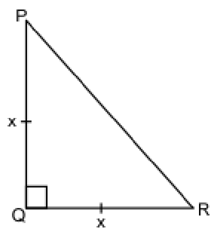
$\therefore$ From (i) and (ii) $PM^2 = QM^2$

$\therefore$ PM = QM

seg PM ≅ seg QM

Q.2 A) Complete the following Activities. (Any two) **(4)**

1) A side of an isosceles right angled triangle is x. Find its hypotenuse.



In △PQR, ∠PQR = 90°

and PQ = QR = x

$\therefore$ PR^2 = ______ ... [Pythagoras theorem]

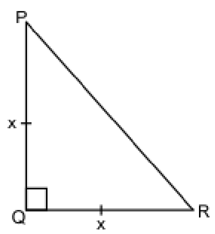
= ______

$\therefore$ PR^2 = ______

$\therefore$ PR = ______units ... [Taking square root]

$\therefore$ The length of hypotenuse is ______units.

Ans. A side of an isosceles right angled triangle is x. Find its hypotenuse.



In △PQR, ∠PQR = 90°

and PQ = QR = x

$\therefore$ $PR^2 = PQ^2 + QR^2$ ... [Pythagoras theorem]

$= x^2 + x^2$

$\therefore$ $PR^2 = 2x^2$

$\therefore$ PR = $\sqrt{2}$x units ... [Taking square root]

$\therefore$ The length of hypotenuse is $\sqrt{2}$ x units.

2) In the figure m (arc LN) = 110°, m (arc PQ) = 50° then complete the following activity to find ∠LMN.

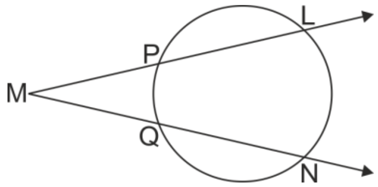
∠LMN = $\frac{1}{2}$ [m (arc LN) - [______]

$\therefore$

$\angle LMN = \frac{1}{2}$ [_____ - 50°]

$\therefore$ $\angle LMN = \frac{1}{2} \times$ _____

$\therefore$ $\angle LMN =$ ______



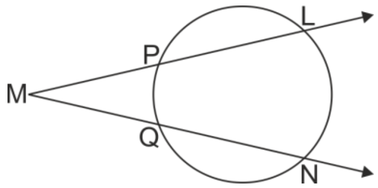
Ans. In the figure m (arc LN) = 110°, m (arc PQ) = 50° then complete the following activity to find $\angle LMN$.

$\angle LMN = \frac{1}{2}$ [m (arc LN) - [m (arc PQ)]

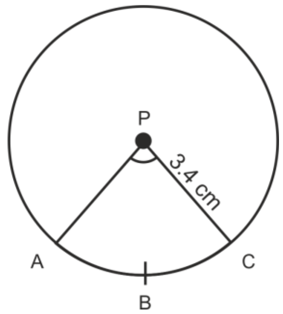
$\therefore$ $\angle LMN = \frac{1}{2}$ [110° - 50°]

$\therefore$ $\angle LMN = \frac{1}{2} \times 60°$

$\therefore$ $\angle LMN = 30°$



3)



In figure, radius of circle is 3.4 cm and perimeter of sector P-ABC is 12.8 cm. Find A(P-ABC).

Given :Radius of circle = r = 3.4 cm

Perimeter of sector = 12.8 cm

:Perimeter = length of the arc + 2r

$\therefore$ Length of arc = 12.8 - 2$\times$3.4

$\therefore$ l = 12.8 - 6.8

$\therefore$ l = ______cm

area of sector = ______

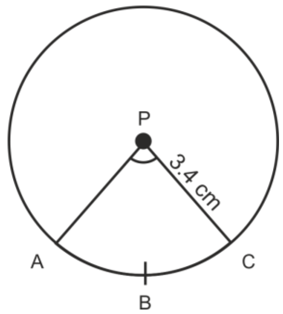
= ______

= 3 × 3.4

=_______cm^2

A(P-ABC) is______ cm^2

Ans.



In figure, radius of circle is 3.4 cm and perimeter of sector P-ABC is 12.8 cm. Find A(P-ABC).

Given :Radius of circle = r = 3.4 cm

Perimeter of sector = 12.8 cm

:Perimeter = length of the arc + 2r

∴ Length of arc = 12.8 - 2×3.4

∴ l = 12.8 - 6.8

∴ l = 6.0 cm

area of sector = $\frac{\text{length of the arc} \times \text{radius}}{2}$

$= \frac{6 \times 3.4}{2}$

$= 3 \times 3.4$

$= 10.2\ cm^2$

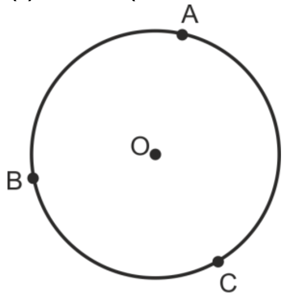
A(P-ABC) is 10.2 cm^2

B) Solve the following questions. (Any four) **(8)**

1) A, B, C are any points on the circle with centre O.

(i) Write the names of all arcs formed due to these points.

(ii) If m arc(BC = 110° and m arc(AB) = 125°, find measures of all remaining arcs.



Ans. (i) Names of arcs -

arc AB, are BC, arc AC, arc ABC, arc ACB, arc BAC

(ii) m(arc ABC) = m(arc AB) + m(arc BC)

= 125° + 110° = 235^0

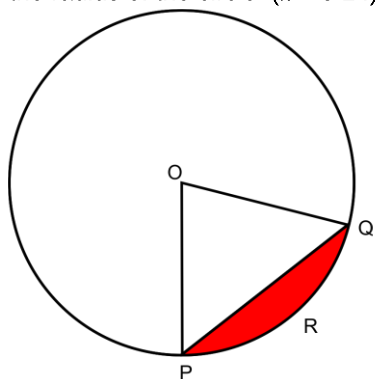
m(arc AC) = 360° − m (arc ACB)

= 360° − 235° = 125°

Similarly, m(arc ACB) = 360° − 125° = 235°

and m(arc BAC) = 360° − 110° = 250°

2) In the figure, if O is the centre of the circle, PQ is a chord. ∠POQ = 90°, area of shaded region is 114 cm^2, find the radius of the circle. (π = 3.14)



Ans. Area of shaded region = $r^2\left(\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right)$

$= r^2\left(\frac{\pi \times 90}{360} - \frac{\sin 90}{2}\right)$ ... {∵ θ = 90° Area of shaded region = 114cm^2}

$114 = r^2\left(\frac{\pi}{4} - \frac{1}{2}\right)$

$114 = r^2\left(\frac{\pi - 2}{4}\right)$

$114 \times 4 = r^2 (1.14)$

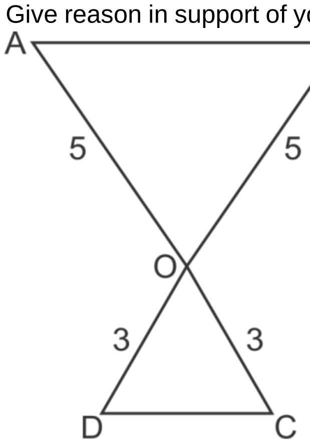
$r^2 = \frac{114 \times 4}{1.14}$

$r^2 = 4 \times 100$

∴ $\boxed{r = 20cm}$

3) In the following figure, indicate whether the triangle are similar or not.

Give reason in support of your answer.



Ans. Proof:

(1) AO = 5, CO = 3 ... (Given)

$\therefore \quad \frac{AO}{CO} = \frac{5}{3}$

(2) BO = 5, DO = 3 ... (Given)

$\therefore \quad \frac{BO}{DO} = \frac{5}{3}$

(3)In △AOB and △COD

For AOB ↔COD

$\frac{AO}{CO} = \frac{BO}{DO}$ ... [From (1) and (2)]

$\therefore$ ∠AOB ≅ ∠COD ... (Vertically opposite angles)

$\therefore$ △AOB ~ △COD ... (S – A – S Test of similarity)

4) If A(3, 5), B(7, 9) and point Q divides seg AB in the ratio 2 : 3 then find co-ordinates of point Q.

Ans. In the given example let $(x_1, y_1) = (3, 5)$

and $(x_2, y_2) = (7, 9)$.

m : n = 2○:3

According to section formula,

$x = \frac{mx_2 + nx_1}{m + n} = \frac{2 \times 7 + 3 \times 3}{2 + 3} = \frac{23}{5}$; $y = \frac{my_2 + ny_1}{m + n} = \frac{2 \times 9 + 3 \times 5}{2 + 3} = \frac{33}{5}$

$\therefore$ Co-ordinates of Q are $\left(\frac{23}{5}, \frac{33}{5}\right)$

5) In △LMN ,l = 5, m = 13, n = 12. State whether △LMN is a right-angled triangle or not.

Ans. LM = 5, MN = 13, LN = 12

$LM^2 = 25$, MN = 169, $LN^2 = 144$

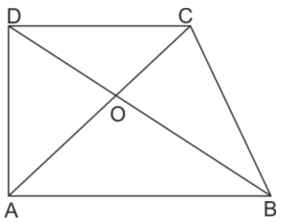
$\therefore$ 169 = 144 + 25

$\therefore$ $MN^2 = LN^2 + LN^2$

$\therefore$ By Converse of Pythagoras theorem △LMN is a right angled triangle.

Q.3 A) Complete the following activity. (Any one) **(3)**

1)



In the given figure, ABCD is a trapezium in which AB ∥ DC. If 2AB = 3DC, find the ratio of the areas of △AOB and △COD.

$\frac{AB}{DC} = \frac{3}{2}$

To find : area △AOB : area of △COD

Proof : In △AOB and △COD

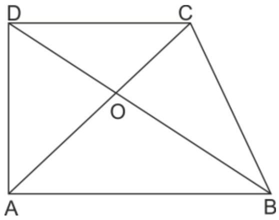
∠AOB = ∠COD ______

∠OAB = ______ (alternate angles)

∴ △AOB ~ △COD ______

∴ $\frac{\text{area } \triangle AOB}{\text{area } \triangle COD}$ = ______ = $\frac{3^2}{2^2}$ = ______

Ratio in the areas of △AOB and △COD______

Ans. 

In the given figure, ABCD is a trapezium in which AB || DC. If 2AB = 3DC, find the ratio of the areas of △AOB and △COD.

$\frac{AB}{DC} = \frac{3}{2}$

To find : area △AOB : area of △COD

Proof : In △AOB and △COD

∠AOB = ∠COD	(vertically opposite angles)
∠OAB = ∠OCD	(alternate angles)
∴ △AOB ~ △COD	(AA axiom)

∴ $\frac{\text{area } \triangle AOB}{\text{area } \triangle COD} = \frac{AB^2}{DC^2} = \frac{3^2}{2^2} = \frac{9}{4}$

Ratio in the areas of △AOB and △COD 9 : 4

2) Some plastic balls of radius 1 cm were melted and cast into a tube. The thickness, length and outer radius of the tube were 2 cm, 90 cm and 30 cm respectively. How many balls were melted to make the tube?

Tube = ______

Radius = ______

∴ = 30 - 2

∴ = ______

Number of balls = ______

∴ $= \frac{\pi(r_1^{\ 2} - r_2^{\ 2})h}{\frac{4}{3}\pi r^3}$

∴ = ______

∴ $= \frac{(900 - 784) \times 90 \times 3}{4}$

∴ = ______

Ans. Some plastic balls of radius 1 cm were melted and cast into a tube. The thickness, length and outer radius of the tube were 2 cm, 90 cm and 30 cm respectively. How many balls were melted to make the tube?

Tube = cylinder

Radius = Outer radius - Thickness

∴ = 30 - 2

∴ = 28cm

Number of balls = $\frac{\text{Volume of tube}}{\text{Volume of ball}}$

∴ $= \frac{\pi(r_1^{\ 2} - r_2^{\ 2})h}{\frac{4}{3}\pi r^3}$

∴ $= \frac{((30)^2 - (28)^2) \times 90}{\frac{4}{3} \times (1)^3}$

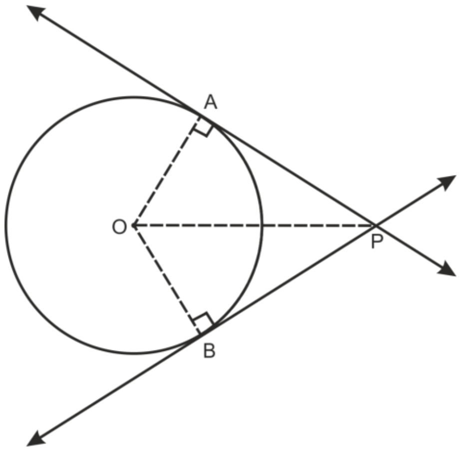
∴ $= \frac{(900 - 784) \times 90 \times 3}{4}$

∴ **= 7830 balls**

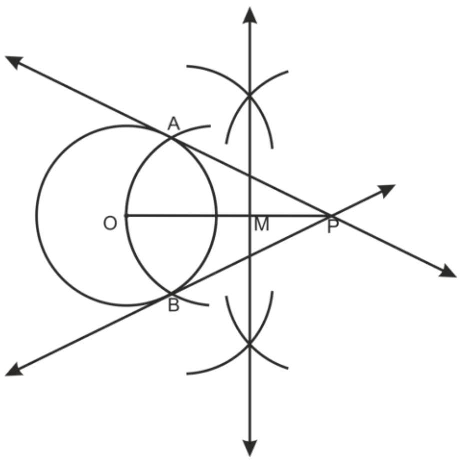
B) Solve the following questions. (Any two) (6)

1) Construct tangents to a circle from a point outside the circle.

Ans.



Let P be a point in the exterior of the circle. Let PA and PB be the tangents to the circle with the centre O, touching the circle in points A and B respectively.



2) If point (x, y) is equidistant from points (7, 1) and (3, 5), show that y = x - 2.

Ans. Let point P (x, y) be equidistant from points A(7, 1) and B(3, 5)

$\therefore$ AP = BP

$\therefore$ $AP^2 = BP^2$

$\therefore$ $(x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$

$\therefore$ $x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$

$\therefore$ $- 8x + 8y = -16$

$\therefore$ $x - y = 2$

$\therefore$ $y = x - 2$

3) Prove that $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

Ans. **Proof:** LHS $= \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,(2\cos^2 - 1)}$

$= \frac{\sin\theta\,(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta - \sin^2\theta - \cos^2\theta)}$ ... $(\sin^2\theta + \cos^2\theta = 1)$

$= \frac{\sin\theta\,(\cos^2\theta - \sin^2\theta)}{\cos\theta\,(\cos^2\theta - \sin^2\theta)}$

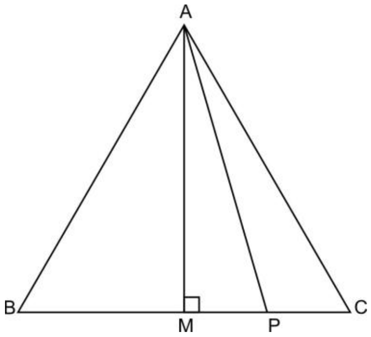
$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$ = RHS

$\therefore$ $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

4) $\triangle$ABC is an equilateral triangle. Point P is on base BC such that PC = $\frac{1}{3}$BC, if AB = 6 cm find AP.

Ans.



Draw seg AM $\perp$ side BC such that B-M-C

$\triangle$ABC is equilateral triangle.

$\therefore$ AB = BC = AC = 6cm ...(Sides of an equilateral triangle) ...(1)

$\angle$C = 60° ...(Sides of an equilateral triangle) ... (2)

In $\triangle$AMC,

$\angle$AMC + $\angle$ACM + $\angle$MAC = 180° ...(Sum of all angles of a triangle is 180°)

$\therefore$ 90° + 60° + $\angle$MAC = 180° ...(From (1) and (2)

$\therefore$ 150 + $\angle$MAC - 180°

$\therefore$ $\angle$MAC = 180° - 150°

$\therefore$ $\angle$MAC = 30°

$\therefore$ $\triangle$AMC is a 30°-60°-90° triangle

$\therefore$ by 30°- 60°-90° triangle theorem

AM = $\frac{\sqrt{3}}{2}$AC ...(Side opposite to 60°)

$\therefore$ AM = $\frac{\sqrt{3}}{2} \times 6$ ...[From (1)]

$\therefore$ AM = $3\sqrt{3}$cm

MC = $\frac{1}{2}$BC ...(Side opposite to 30°)

$\therefore$ MC = $\frac{1}{2} \times 6$ ...[From (1)]

$\therefore$ MC = 3cm

PC = $\frac{1}{3}$BC ...(Given)

$\therefore$ PC = $\frac{1}{3} \times 6$ ...[Fom (1)

$\therefore$ PC = 2cm

MP + PC = MC (M-P-C)

$\therefore$ MP + 2 = 3

$\therefore$ MP = 3 -2

$\therefore$ MP = 1cm

In $\triangle$AMP,

$\angle$AMP - 90° ...(Construction)

$\therefore$ by Pythagoras theorem,

$AP^2 = AM^2 + MP^2$

$\therefore$ $AP^2 = (3\sqrt{3})^2 + 1^2$

$\therefore$ $AP^2 = 27 + 1$

$\therefore$ $AP^2 = 28$

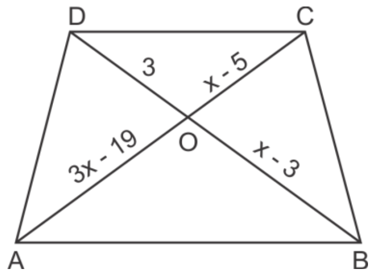
$\therefore$ $AP^2 = 4 \times 7$

$\therefore$ AP = $2\sqrt{7}$cm ...(Taking square roots on both the sides)

$\therefore$ **AP = $2\sqrt{7}$cm**

Q.4 Solve the following questions. (Any two) (8)

1) In the figure, seg AB || seg DC. Using the information given find the value of x.



Ans. (1)In □ ABCD,

seg AB || seg DC ... (given)

∴ $\angle DCA \cong \angle BAO$ ... (Alternate angles for transversal AC)

i.e. $\angle DCO \cong \angle BAO$ ... (A-O-C)

(2)In △DOC and △BOA

(a) $\angle DCO \cong \angle BAO$ ... [from (1)]

(b) $\angle DOC \cong \angle BOA$ ... (vertically opposite angles)

(c) ∴ △DOC ~ △BOA ... (A - A test of similarity)

(3) $\frac{DO}{AO} = \frac{CO}{AO}$ ... (c.s.s.t)

(4) But DO = 3, BO = x - 3

CO = x - 5, AO = 3x - 19 ... (given)

(5) ∴ $\frac{3}{x-3} = \frac{x-5}{3x-19}$ ... [from (3) and (4)]

∴ 3 (3x - 19) = (x - 5) (x - 3)

9x - 57 = x^2 - 8x + 15

∴ x^2 - 8x - 9x + 15 + 57 = 0

∴ x^2 - 17x + 72 = 0

∴ (x - 8) (x - 9) = 0

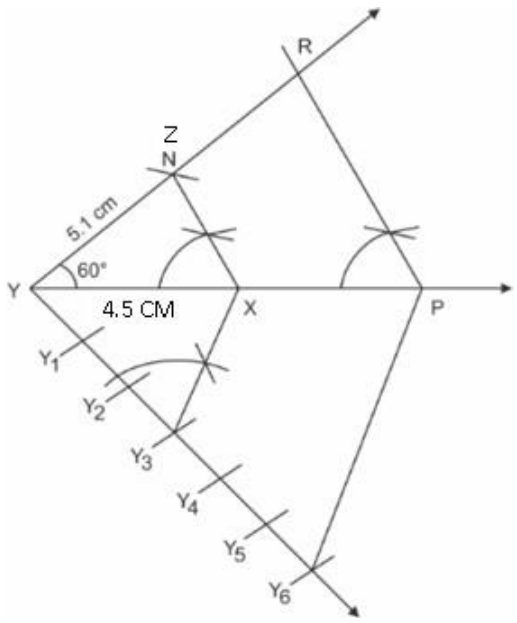
either x - 8 = 0 or x - 9 = 0

∴ x = 8 or x = 9

The value of x = 8 or x = 9

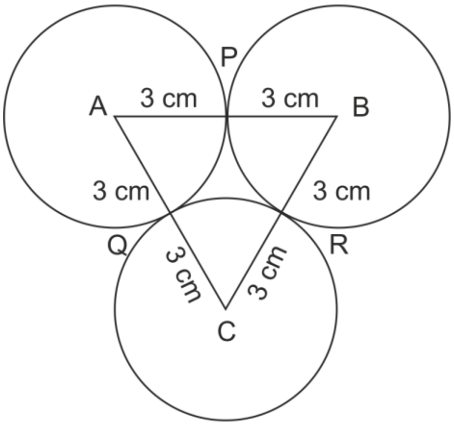
2) △XYZ ~ △PYR; In △XYZ, ∠Y = 60°, XY = 4.5 cm, YZ = 5.1 cm and $\frac{XY}{PY} = \frac{4}{7}$ Construct △XYZ and △PYR.

Ans.



3) Draw circles with centres A, B and C each of radius 3 cm, such that each circle touches the other two circles.

Ans.



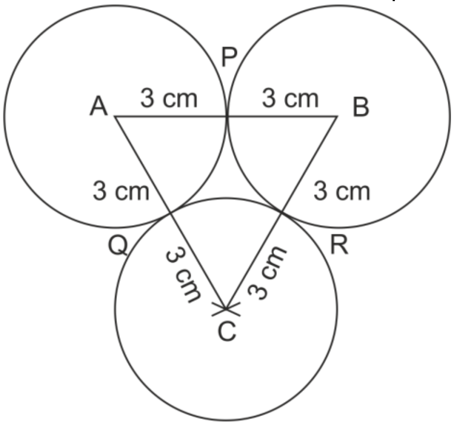
Analysis : Let the circles with centres A, B and C touch at points P, R and Q as shown.
By theorem of touching circles, we get A–P–B, B–R–C and A–Q–C.

$\therefore$ AB = AP + PB = 3 + 3 = 6 cm.

Similarly BC = 6 cm and AC = 6 cm

$\therefore$ We can construct $\triangle$ABC with
AB = BC = AC = 6 cm

With A, B and C as centres, the required circles of radius 3 cm can be drawn.



Q.5 Solve the following questions. (Any one) **(3)**

1) The diameter and length of a roller is 120 cm and 84 cm respectively. To level the ground, 200 rotations of the roller are required. Find the expenditure to level the ground at the rate of Rs. 10 per sq.m.

Ans. Given : diameter of rolles = 120 cm = 1.2m

$\therefore$ radius of roller = 60 cm = 0.6 m

length = height of roller = 84 cm = 0.84m

rate of levelling = Rs. 10 per m^2

No. of rotations = 200

To find : Total cost of levelling

Solution:

Curved Surface Area of Roller = $2\pi rh$

$= 2 \times \frac{22}{7} \times 0.6 \times 0.84$

$= 3.168 m^2$

One rotation of roller will cover $3.168 m^2$ of area, but total rotations 200 are made.

$\therefore$ Total area levelled = 200×3.168 = Rs. $633.6 m^2$

The cost of levelling is Rs. 633.6×10 = Rs. 6336

2) **Prove :** $\frac{\text{cosec } \theta}{\text{cosec } \theta - 1} + \frac{\text{cosec } \theta}{\text{cosec } \theta + 1} = 2 \sec^2 \theta$

Ans. **Proof:** LHS = $\frac{\text{cosec } \theta}{\text{cosec } \theta - 1} + \frac{\text{cosec } \theta}{\text{cosec } \theta + 1}$

$= \frac{\text{cosec } \theta (\text{cosec } \theta + 1) + \text{cosec } \theta (\text{cosec } \theta - 1)}{(\text{cosec } \theta - 1)(\text{cosec } \theta + 1)}$

$= \frac{\text{cosec}^2 \theta + \text{cosec } \theta + \text{cosec}^2 \theta - \text{cosec } \theta}{\text{cosec}^2 \theta - 1}$

$$= \frac{2\ \text{cosec}^2\theta}{\text{cosec}^2 - 1}$$

$$= \frac{2\ (1 + \cot^2\theta)}{1 + \cot^2\theta - 1} \qquad \ldots (\text{cosec}^2\theta = 1 + \cot^2\theta)$$

$$= \frac{2\left(1 + \frac{1}{\tan^2\theta}\right)}{\frac{1}{\tan^2\theta}} \qquad \ldots \left(\cot\theta = \frac{1}{\tan\theta}\right)$$

$$= \frac{2\ (\tan^2 + 1)}{\tan^2\theta} \div \frac{1}{\tan^2\theta}$$

$$= \frac{2\ (\sec^2\theta)}{\tan^2\theta} \times \tan^2\theta \qquad .. (1 + \tan^2\theta = \sec^2\theta)$$

$= 2\sec^2\theta$ = RHS

$$\therefore \quad \frac{\text{cosec}\ \theta}{\text{cosec}\ \theta - 1} + \frac{\text{cosec}\ \theta}{\text{cosec}\ \theta + 1} = 2\sec^2\theta$$